1.3 Using Reasoning to Find a Counterexample to a Conjecture

Counterexample - an example that invalidates a conjecture

In Lesson 1.1, page 9, Francesca and Steffan made conjectures about the difference between consecutive squares.

Steffan's conjecture: The difference between consecutive perfect squares is always an odd number.

Francesca's conjecture: The difference between consecutive perfect squares is always a prime number.

$5^2 - 4^2 = 9$

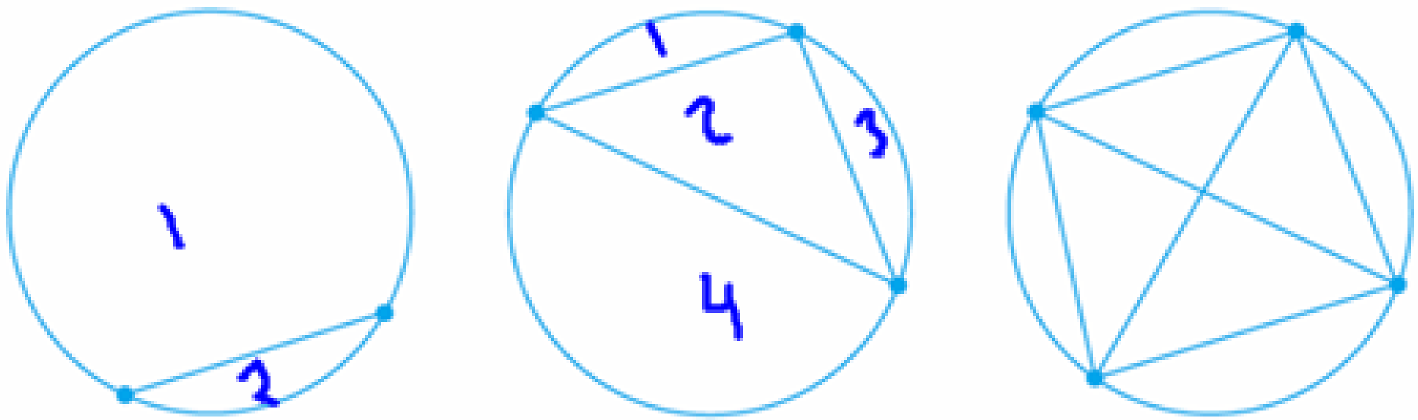
counterexample

How can these conjectures be tested?

→ do a few more tests

→ test anything that is missing

Kerry created a series of circles. Each circle had points marked on its circumference and joined by chords.



As the number of points on the circumference increased, Kerry noticed a pattern for the number of regions created by the chords.

Number of Points	2	3	4
Number of Regions	2	4	8

×2 ×2

. . . . . 5 6
16 32

She made the following conjecture: As the number of connected points on the circumference of a circle increases by 1, the number of regions created within the circle increases by a factor of 2.

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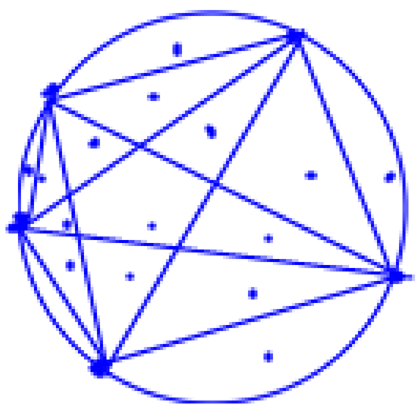

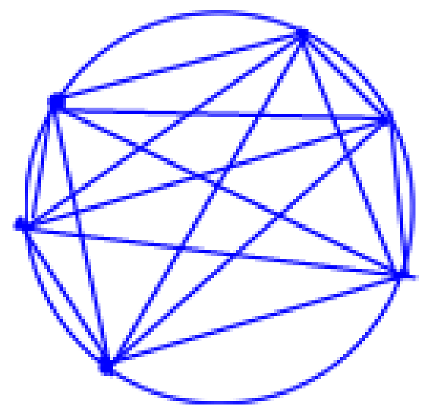

# of points

5

6

# of regions

16 ✓

31

↑ counterexample

Kerry's conjecture is not valid.

EXAMPLE 3 Using reasoning to find a counterexample to a conjecture

Matt found an interesting numeric pattern:

$$1 \cdot 8 + 1 = \mathbf{9}$$
$$12 \cdot 8 + 2 = \mathbf{98}$$
$$123 \cdot 8 + 3 = \mathbf{987}$$
$$1234 \cdot 8 + 4 = \mathbf{9876}$$

Matt thinks that this pattern will continue.

Search for a counterexample to Matt's conjecture.

$12345 \cdot 8 + 5 = 98765$

$123456 \cdot 8 + 6 = 987654$

Kublu's Solution

$$1 \cdot 8 + 1 = 9$$
$$12 \cdot 8 + 2 = 98$$
$$123 \cdot 8 + 3 = 987$$
$$1234 \cdot 8 + 4 = 9876$$

The pattern seemed to be related to the first factor (the factor that wasn't 8), the number that was added, and the product.

	A	B
1	1 · 8 + 1	9
2	12 · 8 + 2	98
3	123 · 8 + 3	987
4	1234 · 8 + 4	9876
5	12345 · 8 + 5	98765
6	123456 · 8 + 6	987654
7	1234567 · 8 + 7	9876543
8	12345678 · 8 + 8	98765432
9	123456789 · 8 + 9	987654321

I used a spreadsheet to see if the pattern continued.

The spreadsheet showed that it did.

counter examples

$$12345678910 \cdot 8 + 10 = 98\,765\,431\,290 \quad \times$$
$$1234567890 \cdot 8 + 10 = 9\,876\,543\,130 \quad \times$$
$$12345678910 \cdot 8 + 0 = 98\,765\,431\,280 \quad \times$$
$$1234567890 \cdot 8 + 0 = 9\,876\,543\,120 \quad \times$$

When I came to the tenth step in the sequence, I had to decide whether to use 10 or 0 in the first factor and as the number to add. I decided to check each way that 10 and 0 could be represented.

The pattern holds true until 9 of the 10 digits are included. At the tenth step in the sequence, a counterexample is found.

Since the pattern did not continue, Matt's conjecture is invalid.

Revise his conjecture: the pattern holds true for numbers 1 to 9.

In Summary

Key Ideas

- Once you have found a counterexample to a conjecture, you have disproved the conjecture. This means that the conjecture is invalid.
- You may be able to use a counterexample to help you revise a conjecture.

Need to Know

- A single counterexample is enough to disprove a conjecture.
- Even if you cannot find a counterexample, you cannot be certain that there is not one. Any supporting evidence you develop while searching for a counterexample, however, does increase the likelihood that the conjecture is true.

Do p.22-23 - #1-9

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