1.4 Proving Conjectures: Deductive Reasoning

deductive reasoning - drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

proofs - a mathematical argument showing that a statement is valid in all cases, or that a counterexample exists.

Jon discovered a pattern when adding integers:

$$1 + 2 + \underline{3} + 4 + 5 = 15 \quad 3.5$$

$$(-15) + (-14) + (-\underline{13}) + (-12) + (-11) = -65 \quad -13.5$$

$$(-3) + (-2) + (\underline{-1}) + 0 + 1 = -5 \quad -1.5$$

He claims that whenever you add five consecutive integers, the sum is always 5 times the median of the numbers.

Inductive Reasoning:

$$16 + 17 + \underline{18} + 19 + 20 = 90 \quad 18.5$$

$$48 + 49 + \underline{50} + 51 + 52 = 250 \quad 50.5$$

Deductive Reasoning:

$$x + (x+1) + (x+2) + (x+3) + (x+4) = 5(x+2)$$

combine like terms

$$5x + 10 = 5x + 10$$

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EXAMPLE 2 Using deductive reasoning to generalize a conjecture

In Lesson 1.3, page 19, Luke found more support for Steffan's conjecture from Lesson 1.1, page 9—that the difference between consecutive perfect squares is always an odd number.

Determine the general case to prove Steffan's conjecture.

$$(x+1)^2 - (x)^2$$

$$\cancel{x^2} + 2x + 1 - \cancel{x^2}$$

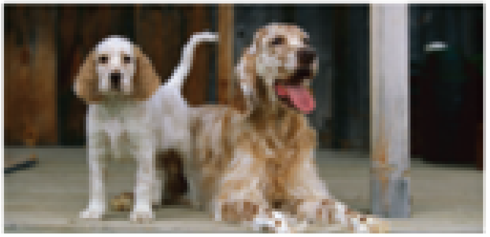
$$2x + 1$$

even $+1$ $\therefore$ odd

EXAMPLE 3 Using deductive reasoning to make a valid conclusion

All dogs are mammals. All mammals are vertebrates. Shaggy is a dog.

What can be deduced about Shaggy?



Shaggy is a mammal; a vertebrate

EXAMPLE 5 Communicating reasoning about a divisibility rule

The following rule can be used to determine whether a number is divisible by 3:

Add the digits, and determine if the sum is divisible by 3. If the sum is divisible by 3, then the original number is divisible by 3.

Use deductive reasoning to prove that the divisibility rule for 3 is valid for two-digit numbers.

ab

$a \cdot 10 + b$

$10a + b$

$9a + a + b$

multiple of 3

If $a+b$ is divisible by 3 then ab is divisible by 3

Use deductive reasoning to prove the following conjecture.

"the sum of any three consecutive integers is a multiple of 3"

$x, x+1, x+2$

$x + x+1 + x+2$

$3x + 3$

$3(x+1)$

multiple of 3.

When two odd numbers are added, their sums are always even.

$2x+1$ & $2y+1$

$2x+1+2y+1$

$2x+2y+2$

$2(x+y+1)$

? multiple of 2 $\therefore$ even

7. Drew created this step-by-step number trick:
   - Choose any number.
   - Multiply by 4.
   - Add 10.
   - Divide by 2.
   - Subtract 5.
   - Divide by 2.
   - Add 3.

   a) Show inductively, using three examples, that the result is always 3 more than the chosen number.

   b) Prove deductively that the result is always 3 more than the chosen number.

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$7 \to 28 \to 38 \to 19 \to 14 \to 7 \to 10$

+3

$7+3=10$

$2 \to 8 \to 18 \to 9 \to 4 \to 2 \to 5$

+3

$2+3=5$

7. Drew created this step-by-step number trick:

- Choose any number.
- Multiply by 4.
- Add 10.
- Divide by 2.
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- Add 3.

a) Show inductively, using three examples, that the result is always 3 more than the chosen number.

b) Prove deductively that the result is always 3 more than the chosen number.

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$x \to 4x \to 4x+10 \to 2x+5 \to 2x \to x \to x+3$

$+3$

Do p. 31-33 - #4, 5, 8-11, 14, 16