6.5 Optimization Problems II: Exploring Solutions

1. Where might you find the maximum and minimum solutions to each objective function below? Explain how you know.

a) Model A

Restrictions:

$x \in \mathrm{R}, y \in \mathrm{R}$

Constraints:

$x > -4$
$x - y \leq 8$
$y \leq 3$

Graph → find vertices

Objective function:

$T = 2x + 5y$

sub in

find max & min

a) Model A

Restrictions:

$x \in \mathrm{R}, y \in \mathrm{R}$

Constraints:

$x > -4$

$x - y \leq 8$

$y \leq 3$

Objective function:

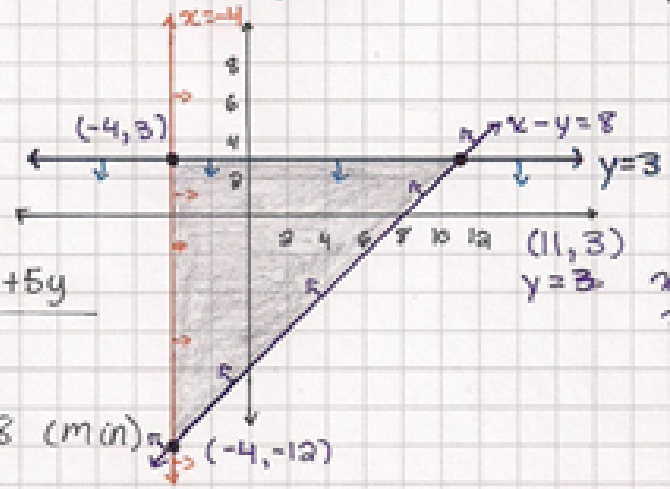
$T = 2x + 5y$

1a. $x - y \leq 8$ $\quad x > -4$ $\quad y \leq 3$ $\quad T = 2x + 5y$

$x - y = 8$

x	y
0	8
8	0

$0 \leq 8$ ✓



Vertices	$T = 2x + 5y$
(-4, 3)	7
(-4, -12)	-68 (min)
(11, 3)	37 (max)

$y = 3$

$x - y = 8$

$x - 3 = 8$

$x = 11$

$x = -4$

$x - y = 8$

$-4 - y = 8$

$-y = 12$

$y = -12$

1. Where might you find the maximum and minimum solutions to each objective function below? Explain how you know.

b) Model B

Restrictions:

$x \in W, y \in W$

Constraints:

$x \geq 0$

$y \geq 0$

$y > 4$

$y + x \leq 9$

Objective function:

$N = 3x - 2y$

1b. $x \geq 0$ $y \geq 0$ $y > 4$ $x + y \leq 9$ $N = 3x - 2y$

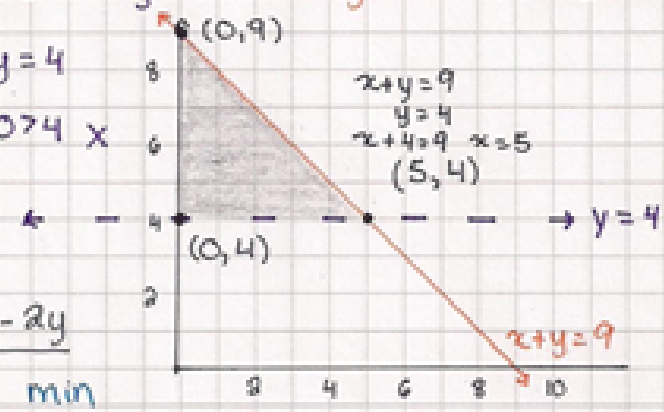
$x + y = 9$

x	y
0	9
9	0

$0 \leq 9$ ✓

$y = 4$

$0 > 4$ X



Vertices	$N = 3x - 2y$
(0,9)	−18 min
(0,4)	−8
(5,4)	7 max

2. Consider the model below. What point in the feasible region would result in the minimum value of the objective function? How could you have predicted this from examining the objective function?

Restrictions:
$x \in \mathrm{R}, y \in \mathrm{R}$

Constraints:
$x + 4y \leq 12$
$x - y \leq 2$
$x \geq -4$

Objective function:
$P = x - y$

2. $x+4y \leq 12$ $\quad$ $x-y \leq 2$ $\quad$ $x \geq -4$ $\quad$ $P = x - y$

$x+4y=12$

x	y
0	3
12	0

$0 \leq 12$ ✓

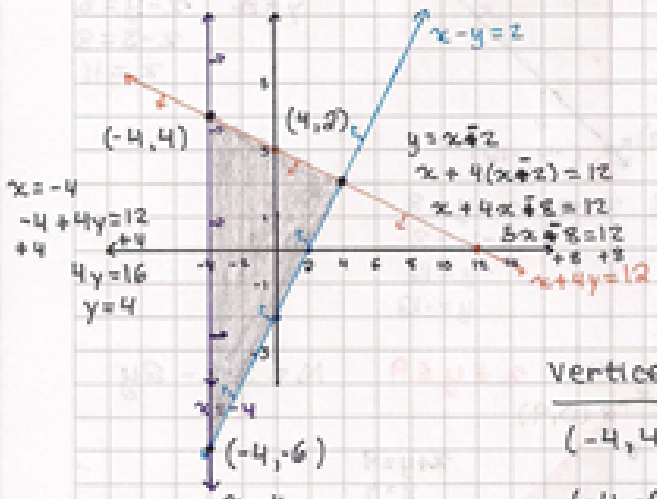
$x-y=2$

x	y
0	-2
2	0

$0 \leq 2$ ✓

$x=-4$

$0 \geq -4$ ✓



$y = x - 2$
$x + 4(x-2) = 12$
$x + 4x - 8 = 12$
$5x - 8 = 12$
$+8 \quad +8$
$5x = 20$
$x = 4$

$y = 4 - 2$
$y = 2$

$x = -4$
$-4 + 4y = 12$
$+4 \quad +4$
$4y = 16$
$y = 4$

$x = -4$
$x - y = 2$
$-4 - y = 2$
$+4 \quad +4$
$-y = 6$
$y = -6$

Vertices	$P = x - y$	
$(-4, 4)$	$P = (-4) - 4 = -8$	min.
$(-4, -6)$	$P = (-4) - (-6) = 2$	max
$(4, 2)$	$P = 4 - 2 = 2$	max

3. Meg is building a bookshelf to display her cookbooks and novels.

- She has no more than 50 cookbooks and no more than 200 novels.
- She wants to display at least 2 novels for every cookbook.
- The cookbook spines are about half an inch wide, and the novel spines are about a quarter of an inch wide.

Meg wants to know how long to make the bookshelf.

The following model represents this situation.

Let c represent the number of cookbooks.
Let n represent the number of novels.
Let W represent the width of the bookshelf.

Restrictions:
$c \in W$, $n \in W$

Constraints:
$c \geq 0$
$n \geq 0$
$c \leq 50$
$n \leq 200$
$n \geq 2c$

Objective function:
$W = 0.5c + 0.25n$

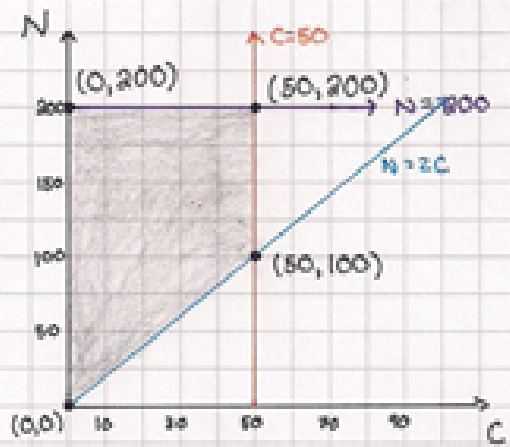
3. C: cookbooks C, N ∈ W W = 0.5C + 0.25N
N: novels

C ≤ 50 N ≤ 200 N ≥ 2C

C = 50 N = 200 N = 2C check (100, 100)

N	C
0	0
50	100

0 ≤ 50 ✓ 0 ≤ 200 ✓ 100 ≥ 200 ✗



Vertices	W = 0.5C + 0.25N
(0,0)	W = 0 min
(0,200)	W = 50
(50,100)	W = 50
(50,200)	W = 75 max

a) (50, 200) : 50 cookbooks, 200 novels } 250 books
b) No. N ≥ 2C
c) (50,100) : 50 cookbooks, 100 novels
d) (50,200) : 75 inches
e) (0,0): no shelf, no books