

Quadratic Functions

7.1

Exploring Quadratic Relations

The general form of a quadratic is

$$y = ax^2 + bx + c$$

$a \neq 0$ in degree 2 polynomial

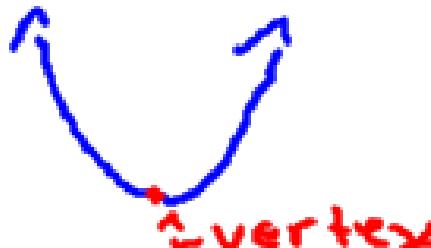
degree of a polynomial = highest power of x

$$y = 3x + 7 - \text{degree 1}$$

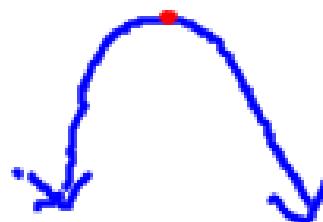
$$y = 2x^3 + 3x^2 + 4x + 5 - \text{degree 3}$$

$$y = ax^2 + bx + c$$

The graph of a quadratic is
a parabola



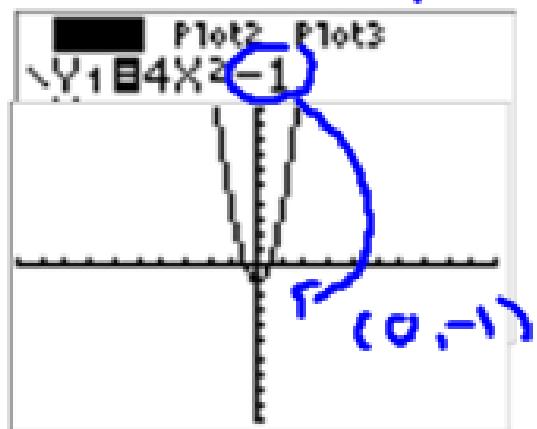
- concave up
- "opens" up
- 'a' is positive
 $a > 0$
- vertex is a minimum



- concave down
- "opens" down
- 'a' is negative
 $a < 0$
- vertex is a maximum

'a'

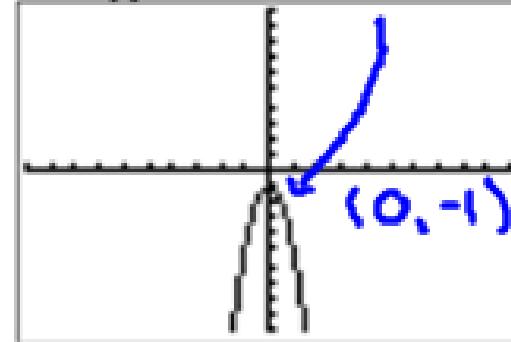
x^2 is positive
 \therefore concave up



x^2 is negative
 \therefore concave down

Plot1 Plot2 Plot3

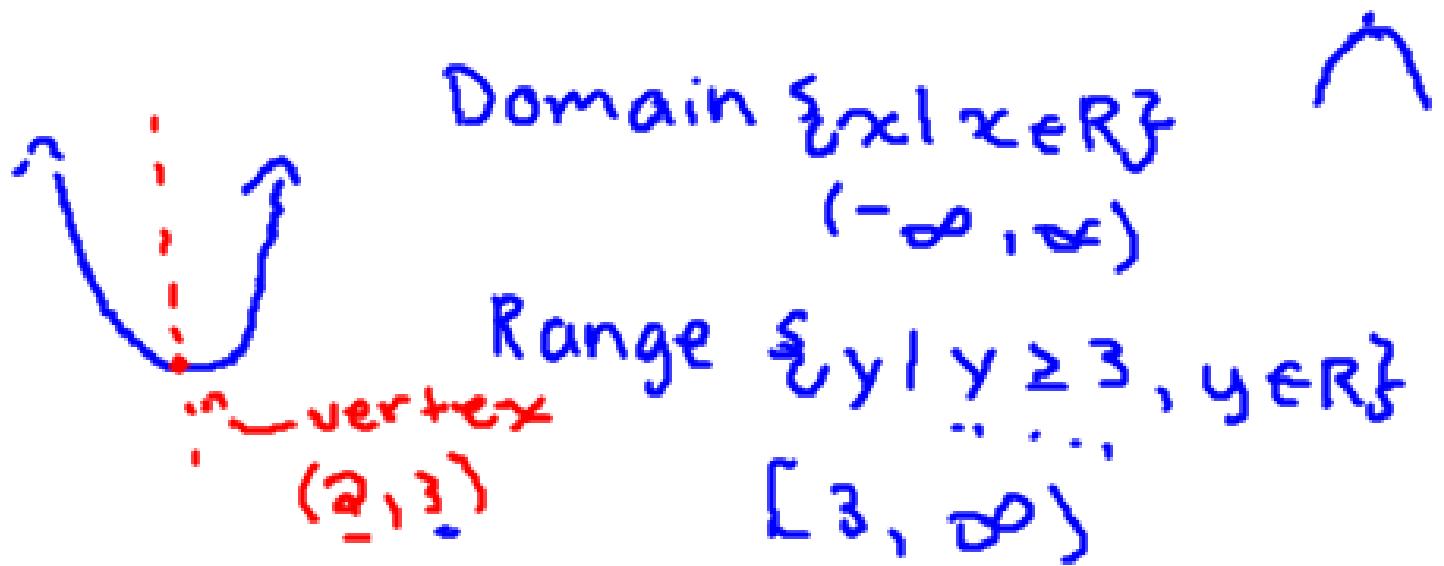
$$Y_1 = -4X^2 - 1$$



$$y = ax^2 + bx + c$$

\uparrow y-int.

$$y = 2x^2 + 3x + 4 \quad \text{y-int } (0, 4)$$
$$y = 3x^2 - 2x - 10 \quad \text{y-int } (0, -10)$$



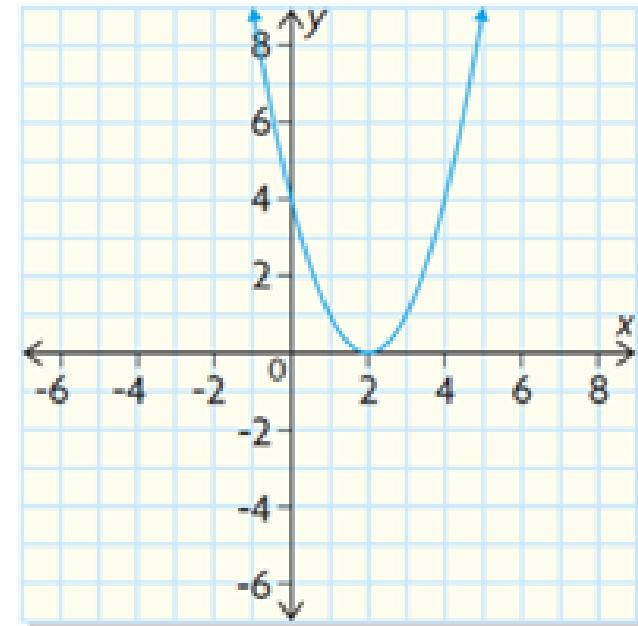
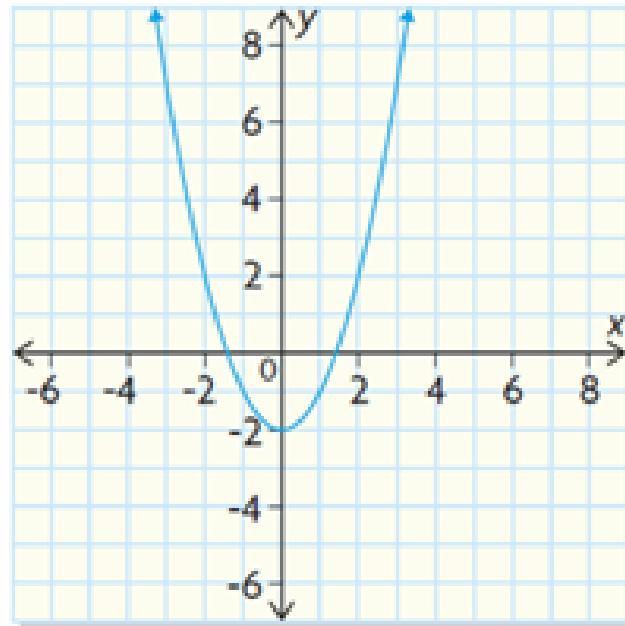
axis of symmetry goes through
 the vertex

axis of symmetry $x = 2$

y-value of vertex is:

- the minimum (concave up)
- the maximum (concave down)

$(2, 3)$ $\{y \mid y \leq 3, y \in \mathbb{R}\}$
 $(-\infty, 3]$



$a > 0$, concave up
 minimum $(0, -2)$
 $y\text{-int } (0, -2)$

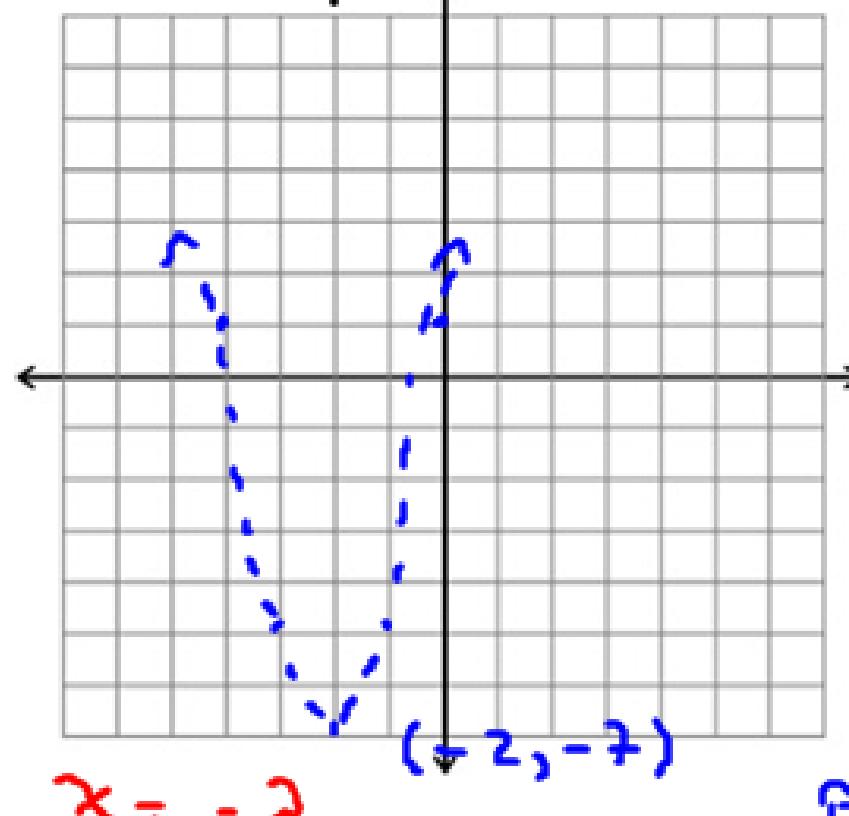
Domain: $\{x \mid x \in \mathbb{R}\}$ $(-\infty, \infty)$

Range: $\{y \mid y \geq -2, y \in \mathbb{R}\}$ $[-2, \infty)$

$$y = 2x^2 + 8x + 1$$

$$y = 2(-4)^2 + 8(-4) + 1$$

$$y = 32 + (-32) + 1 \quad y = 1$$



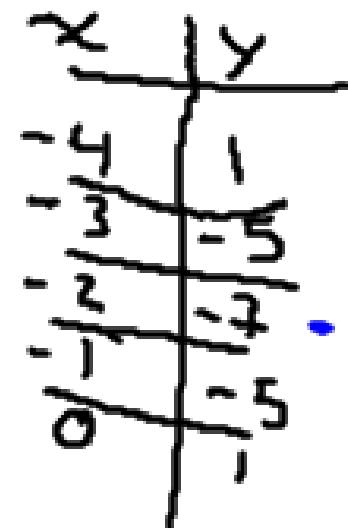
$$x = -2$$

$$(-2, -7)$$

Range $[-7, \infty)$

$$\{y \mid y \geq -7, y \in \mathbb{R}\}$$

$$\text{Pr. } 360 \\ \text{At } 1-6$$



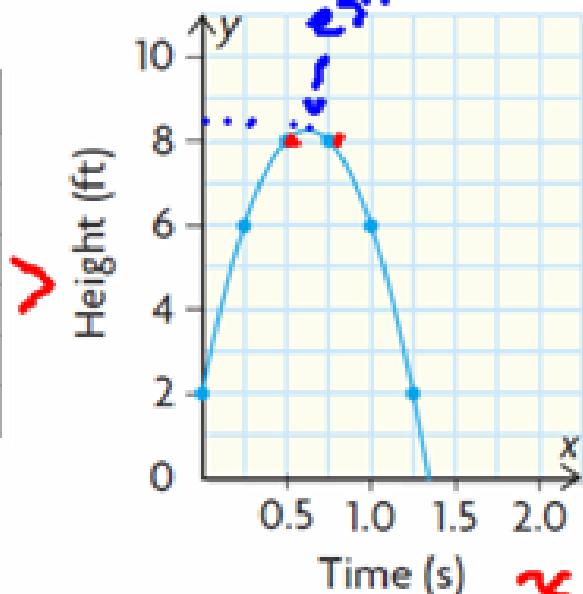
7.2

Properties of Graphs of Quadratic Functions

EXAMPLE 1

Using symmetry to estimate the coordinates of the vertex

Time (s)	Height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2



(x, y)

2 points at same height

$(0.50, 8)$ and $(0.75, 8)$

$$x = \frac{0.50 + 0.75}{2} = 0.625$$

$$x \approx 0.625$$

$$\text{vertex } (0.625, 8.2)$$

Q. 363

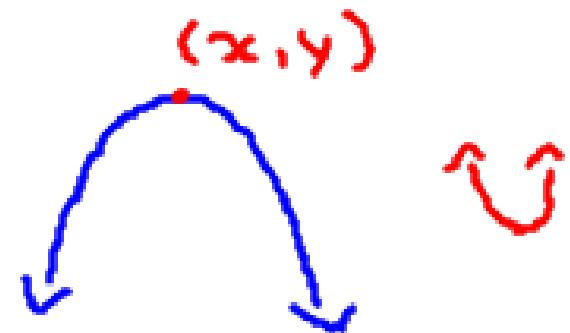
EXAMPLE 2

Reasoning about the maximum value of a quadratic function

Some children are playing at the local splash pad. The water jets spray water from ground level. The path of water from one of these jets forms an arch that can be defined by the function

$$f(x) = -0.12x^2 + 3x$$

where x represents the horizontal distance from the opening in the ground in feet and $f(x)$ is the height of the sprayed water, also measured in feet. What is the maximum height of the arch of water, and how far from the opening in the ground can the water reach?



max height:
18.75 ft.

x	0	1	2	}	12	13
$f(x)$	0	2.88	5.52		18.72	18.72



vertex.

$$x = \frac{12+13}{2} = 12.5 \quad (12.5, 18.75)$$

$$f(12.5) = -0.12(12.5)^2 + 3(12.5)$$

$$= -18.75 + 37.5$$

$$y = 18.75$$

EXAMPLE 3

Graphing a quadratic function using a table of values

Sketch the graph of the function:

$$y = x^2 + x - 2$$

$$\rightarrow y = (-3)^2 + (-3) - 2$$

$$= 9 + (-5)$$

$$y = 4$$

Determine the y -intercept, any x -intercepts, the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function.

Domain $(-\infty, \infty)$

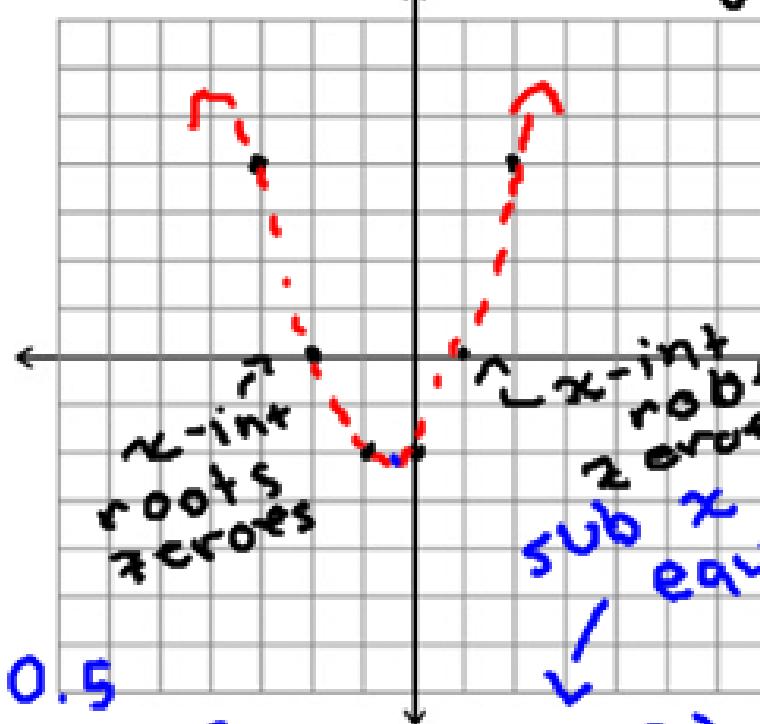
Range $[-2.25, \infty)$

x	y
-3	4
-2	0
-1	-2
0	-2
1	0
2	4

vertex:

$$x = \frac{-1+0}{2} = -0.5$$

$$(-0.5, -2.25)$$



into
equation

EXAMPLE 4**Locating a vertex using technology**

A skier's jump was recorded in frame-by-frame analysis and placed in one picture, as shown.

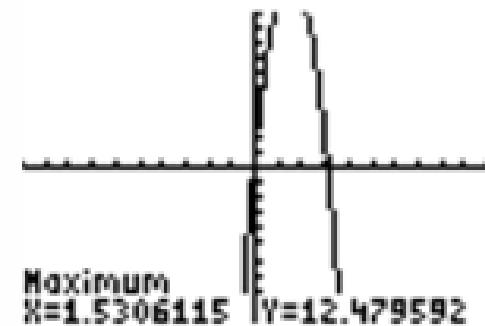


The skier's coach used the picture to determine the quadratic function that relates the skier's height above the ground, y , measured in metres, to the time, x , in seconds that the skier was in the air:

$$y = -4.9x^2 + 15x + 1$$

x, T, Θ, n

Graph the function. Then determine the skier's maximum height, to the nearest tenth of a metre, and state the range of the function for this context.



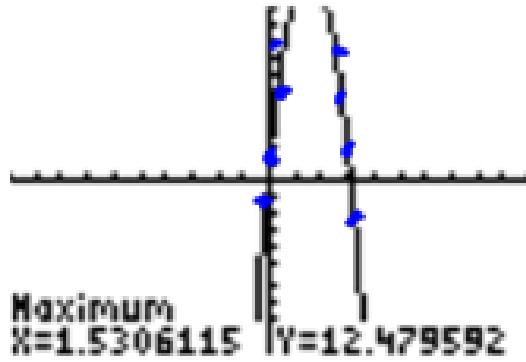
Graph

$y =$

$$-4.9x^2 + 15x + 1$$

GRAPH

zoom \rightarrow 6:2 standard



find max/min vertex
(1.53, 12.50)

[**2ND**] → [**TRACE**]

→ 4: max

Left bound? Enter

Right bound? Enter

Guess? Enter

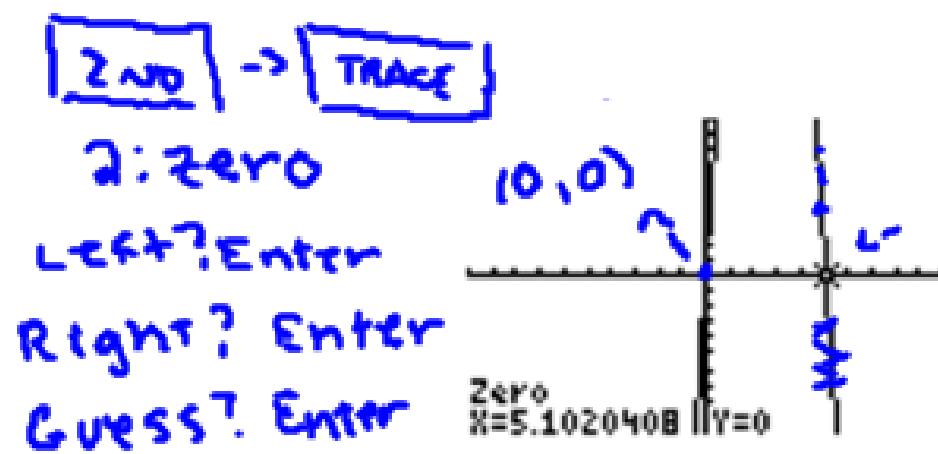
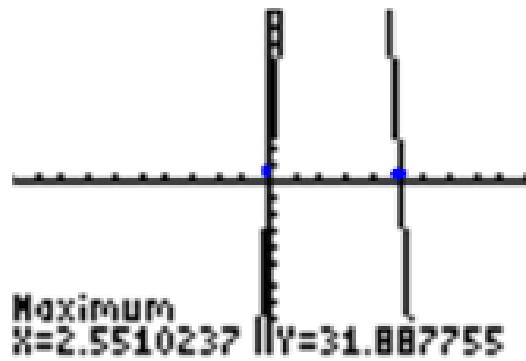
13. In the game of football, a team can score by kicking the ball over a bar and between two uprights. For a kick in a particular game, the height of the ball above the ground, y , in metres, can be modelled by the function

$$y = -4.9x^2 + 25x$$

where x is the time in seconds after the ball left the foot of the player.

- a) Determine the maximum height that this kick reached, to the nearest tenth of a metre. $y = 31.9 \text{ m}$
- b) State any restrictions that the context imposes on the domain and range of the function. $x \geq 0, 0 \leq y \leq 31.9$
- c) How long was the ball in the air?

5.10 s



$\approx 5.10 \text{ s}$