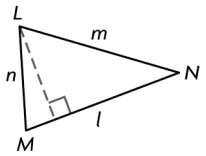
Chapter 3: Acute Triangle Trigonometry

Lesson 3.1: Exploring Side–Angle Relationships in Acute Triangles, page 117

1. a) i)



ii) Let h represent the height of the triangle.
In the small right triangle, In the large right triangle,

$$\sin M = \frac{h}{n} \qquad \sin N = \frac{h}{m}$$

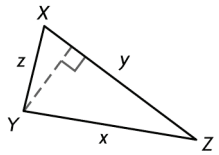
$$n(\sin M) = n\left(\frac{h}{n}\right) \qquad m(\sin N) = m\left(\frac{h}{m}\right)$$

$$h = n \sin M \qquad h = m \sin N$$

$$n \sin M = m \sin N$$

$$\frac{n}{\sin N} = \frac{m}{\sin M}$$

b) i)



ii) Let h represent the height of the triangle.
In the large right triangle, In the small right triangle,

$$\sin Z = \frac{h}{x} \qquad \sin X = \frac{h}{z}$$

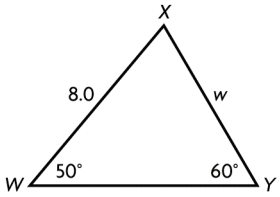
$$x \sin Z = x\left(\frac{h}{x}\right) \qquad z \sin X = z\left(\frac{h}{z}\right)$$

$$h = x \sin Z \qquad h = z \sin X$$

$$x \sin Z = z \sin X$$

$$\frac{x}{\sin X} = \frac{z}{\sin Z}$$

2. a) i)



ii)

$$\frac{w}{\sin 50°} = \frac{8.0}{\sin 60°}$$

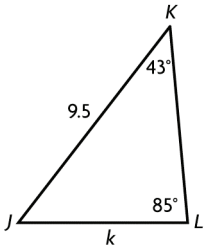
$$\sin 50°\left(\frac{w}{\sin 50°}\right) = \sin 50°\left(\frac{8.0}{\sin 60°}\right)$$

$$w = \frac{8.0(\sin 50°)}{\sin 60°}$$

$$w = 7.076...$$

The length of w is 7.1 units.

b) i)



ii)

$$\frac{k}{\sin 43°} = \frac{9.5}{\sin 85°}$$

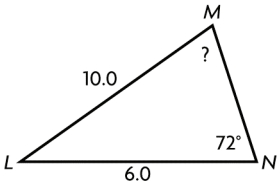
$$\sin 43°\left(\frac{k}{\sin 43°}\right) = \frac{9.5(\sin 43°)}{\sin 85°}$$

$$k = \frac{9.5(\sin 43°)}{\sin 85°}$$

$$k = 6.503...$$

The length of k is 6.5 units.

c) i)



ii)

$$\frac{\sin M}{6.0} = \frac{\sin 72°}{10.0}$$

$$6.0\left(\frac{\sin M}{6.0}\right) = 6.0\left(\frac{\sin 72°}{10.0}\right)$$

$$\sin M = 6.0\left(\frac{\sin 72°}{10.0}\right)$$

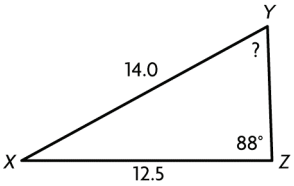
$$\sin M = 0.5706...$$

$$\angle M = \sin^{-1}(0.5706...)$$

$$\angle M = 34.794...°$$

The measure of $\angle M$ is 34.8°.

d) i)



ii) $$\frac{\sin Y}{12.5} = \frac{\sin 88°}{14.0}$$

$$12.5\left(\frac{\sin Y}{12.5}\right) = 12.5\left(\frac{\sin 88°}{14.0}\right)$$

$$\sin Y = 12.5\left(\frac{\sin 88°}{14.0}\right)$$

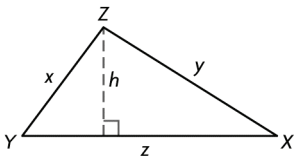
$$\sin Y = 0.8923...$$

$$\angle Y = \sin^{-1}(0.8923...)$$

$$\angle Y = 63.165...°$$

The measure of $\angle Y$ is 63.2°.

3. Agree.



In the large right triangle,

$$\sin X = \frac{h}{y}$$

$$y \sin X = y\left(\frac{h}{y}\right)$$

$$h = y \sin X$$

In the small right triangle,

$$\sin Y = \frac{h}{x}$$

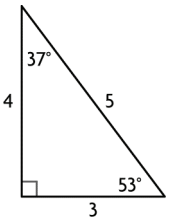
$$x \sin Y = x\left(\frac{h}{x}\right)$$

$$h = x \sin Y$$

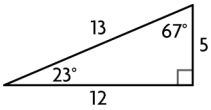
$\therefore\ y \sin X = x \sin Y$

4. e.g., You need the lengths of two sides and the measure of the angle opposite one of the sides or the measures of two angles and the length of any side.

5. e.g., Yes, the ratios are equivalent. Answers have been rounded to the nearest whole number.



$$\frac{3}{\sin 37°} = 5 \quad \frac{4}{\sin 53°} = 5 \quad \frac{5}{\sin 90°} = 5$$



$$\frac{5}{\sin 23°} = 13 \quad \frac{12}{\sin 67°} = 13 \quad \frac{13}{\sin 90°} = 13$$

Lesson 3.2: Proving and Applying the Sine Law, page 124

1. $\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{s}{\sin S}$ or

$$\frac{\sin Q}{q} = \frac{\sin R}{r} = \frac{\sin S}{s}$$

2. a) $$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 72°} = \frac{27.2}{\sin 43°}$$

$$\sin 72°\left(\frac{b}{\sin 72°}\right) = \sin 72°\left(\frac{27.2}{\sin 43°}\right)$$

$$b = \sin 72°\left(\frac{27.2}{\sin 43°}\right)$$

$$b = 37.930...$$

The length of b is 37.9 cm.

b) $$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{37.1} = \frac{\sin 44°}{29.5}$$

$$37.1\left(\frac{\sin \theta}{37.1}\right) = 37.1\left(\frac{\sin 44°}{29.5}\right)$$

$$\sin \theta = 0.8736...$$

$$\theta = \sin^{-1}(0.8736...)$$

$$\theta = 60.882...°$$

The measure of θ is 61°.

3. a) $\angle F + 53° + 68° = 180°$

$$\angle F = 59°$$

$$\frac{d}{\sin D} = \frac{f}{\sin F}$$

$$\frac{d}{\sin 53°} = \frac{22.5}{\sin 59°}$$

$$\sin 53°\left(\frac{d}{\sin 53°}\right) = \sin 53°\left(\frac{22.5}{\sin 59°}\right)$$

$$d = \sin 53°\left(\frac{22.5}{\sin 59°}\right)$$

$$d = 20.963...$$

The length of d is 21.0 cm.

b) $\angle C + 40° + 60° = 180°$

$$\angle C = 80°$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 40°} = \frac{40.0}{\sin 80°}$$

$$\sin 40°\left(\frac{a}{\sin 40°}\right) = \sin 40°\left(\frac{40.0}{\sin 80°}\right)$$

$$a = \sin 40°\left(\frac{40.0}{\sin 80°}\right)$$

$$a = 26.108...$$