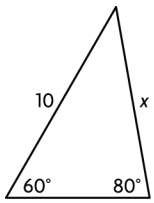
Mid-Chapter Review, page 129

1. $\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$ or

$\frac{\sin X}{x} = \frac{\sin Y}{y} = \frac{\sin Z}{z}$

2. a) e.g.,



b)

$$\frac{x}{\sin 60°} = \frac{10}{\sin 80°}$$

$$\sin 60°\left(\frac{x}{\sin 60°}\right) = \sin 60°\left(\frac{10}{\sin 80°}\right)$$

$$x = \sin 60°\left(\frac{10}{\sin 80°}\right)$$

$$x = 8.793...$$

The length of x is 8.8 units.

3. e.g., Disagree. You cannot rearrange Nazir's expression so that f and $\sin F$ are in one ratio and d and $\sin D$ are in the other.

4. a) $\frac{\sin\theta}{4.0} = \frac{\sin 50°}{4.5}$

$$\sin\theta = 4.0\left(\frac{\sin 50°}{4.5}\right)$$

$$\theta = \sin^{-1}(0.6809...)$$

$$\theta = 42.916...°$$

The measure of θ is 42.9°.

Let X represent the measure of the angle opposite side x.

$$\angle X + 50° + \angle\theta = 180°$$

$$\angle X + 50° + 42.9° = 180°$$

$$\angle X = 87.1°$$

$$\frac{x}{\sin X} = \frac{4.5}{\sin 50°}$$

$$\sin 87.1°\left(\frac{x}{\sin 87.1°}\right) = \sin 87.1°\left(\frac{4.5}{\sin 50°}\right)$$

$$x = \sin 87.1°\left(\frac{4.5}{\sin 50°}\right)$$

$$x = 5.866...$$

The length of x is 5.9 cm.

b)

$$\theta + 55° + 63° = 180° \quad \text{Sum of interior angles of triangle}$$

$$\theta = 62°$$

The measure of θ is 62.0°.

$$\frac{x}{\sin 63°} = \frac{10.5}{\sin\theta}$$

$$\frac{x}{\sin 63°} = \frac{10.5}{\sin 62°}$$

$$\sin 63°\left(\frac{x}{\sin 63°}\right) = \sin 63°\left(\frac{10.5}{\sin 62°}\right)$$

$$x = \sin 63°\left(\frac{10.5}{\sin 62°}\right)$$

$$x = 10.595...$$

The length of x is 10.6 cm.

$$\frac{y}{\sin 55°} = \frac{10.5}{\sin\theta}$$

$$\frac{y}{\sin 55°} = \frac{10.5}{\sin 62°}$$

$$\sin 55°\left(\frac{y}{\sin 55°}\right) = \sin 55°\left(\frac{10.5}{\sin 62°}\right)$$

$$y = \sin 55°\left(\frac{10.5}{\sin 62°}\right)$$

$$y = 9.741...$$

The length of y is 9.7 cm.

5. a) $\angle C + \angle A + \angle B = 180°$

$$\angle C + 70° + 50° = 180°$$

$$\angle C = 60°$$

The measure of $\angle C$ is 60.0°.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 50°} = \frac{15}{\sin 70°}$$

$$\sin 50°\left(\frac{b}{\sin 50°}\right) = \sin 50°\left(\frac{15}{\sin 70°}\right)$$

$$b = \sin 50°\left(\frac{15}{\sin 70°}\right)$$

$$b = 12.228...$$

The length of b is 12.2 cm.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 60°} = \frac{15}{\sin 70°}$$

$$\sin 60°\left(\frac{c}{\sin 60°}\right) = \sin 60°\left(\frac{15}{\sin 70°}\right)$$

$$c = \sin 60°\left(\frac{15}{\sin 70°}\right)$$

$$c = 13.8240...$$

The length of c is 13.8 cm.

b) $\angle L + \angle M + \angle N = 180°$

$\angle L + 81° + 14° = 180°$

$\angle L = 85°$

The measure of $\angle L$ is 85.0°.

$$\frac{m}{\sin M} = \frac{n}{\sin N}$$

$$\frac{m}{\sin 81°} = \frac{8}{\sin 14°}$$

$$\sin 81°\left(\frac{m}{\sin 81°}\right) = \sin 81°\left(\frac{8}{\sin 14°}\right)$$

$$m = \sin 81°\left(\frac{8}{\sin 14°}\right)$$

$$m = 32.661...$$

The length of m is 32.7 cm.

$$\frac{l}{\sin L} = \frac{n}{\sin N}$$

$$\frac{l}{\sin 85°} = \frac{8}{\sin 14°}$$

$$\sin 85°\left(\frac{l}{\sin 85°}\right) = \sin 85°\left(\frac{8}{\sin 14°}\right)$$

$$l = \sin 85°\left(\frac{8}{\sin 14°}\right)$$

$$l = 32.942...$$

The length of l is 32.9 cm.

6. e.g., The measure of angle X or of angle Z is needed to solve the triangle.

7. a) e.g., The shortest distance is across from the smallest angle. Therefore, the tower at B is closer to the fire.

b) Let C represent the measure of remaining unknown angle.

$\angle C + \angle A + \angle B = 180°$

$\angle C + 53° + 65° = 180°$

$\angle C = 62°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 53°} = \frac{3.4}{\sin 62°}$$

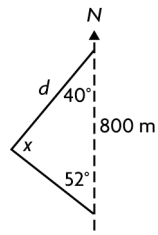
$$\sin 53°\left(\frac{a}{\sin 53°}\right) = \sin 53°\left(\frac{3.4}{\sin 62°}\right)$$

$$a = \sin 53°\left(\frac{3.4}{\sin 62°}\right)$$

$$a = 3.075...$$

The shortest distance from a tower to the fire is 3.1 km.

8.



$x + 40° + 52° = 180°$

$x = 88°$

$$\frac{d}{\sin 52°} = \frac{800}{\sin x}$$

$$\sin 52°\left(\frac{d}{\sin 52°}\right) = \sin 52°\left(\frac{800}{\sin 88°}\right)$$

$$d = \sin 52°\left(\frac{800}{\sin 88°}\right)$$

$$d = 630.792...$$

The distance to the campsite is 631 m.

9. a) $\angle E = \angle F$ Isosceles triangle

$\angle E + \angle F + 50° = 180°$

$2\angle E = 130°$

$\angle E = 65°, \angle F = 65°$

$$\frac{e}{\sin E} = \frac{d}{\sin D}$$

$$\frac{e}{\sin 65°} = \frac{25}{\sin 50°}$$

$$\sin 65°\left(\frac{e}{\sin 65°}\right) = \sin 65°\left(\frac{25}{\sin 50°}\right)$$

$$e = \sin 65°\left(\frac{25}{\sin 50°}\right)$$

$$e = 29.577...$$

Perimeter = $d + e + f$

Perimeter = 25 + 29.577… + 29.577…

Perimeter = 84.155…

The perimeter is 84.2 cm.

b) $\angle O + 55° + 55° = 180°$

$\angle O = 70°$

$$\frac{p}{\sin P} = \frac{o}{\sin O}$$

$$\frac{p}{\sin 55°} = \frac{30}{\sin 70°}$$

$$\sin 55°\left(\frac{p}{\sin 55°}\right) = \sin 55°\left(\frac{30}{\sin 70°}\right)$$

$$p = \sin 55°\left(\frac{30}{\sin 70°}\right)$$

$$p = 26.151...$$

$$q = p$$

Perimeter = $o + p + q$

Perimeter = 30 + 26.151… + 26.151…

Perimeter = 82.303… cm

The perimeter is 82.3 cm.